2D Resistivity Reconstruction via FEM-Based Bayesian Method

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Abstract — The resistivity reconstruction method is studied in this paper. The prior of "blocky" resistivity profile is formulated based on the minimal of total variation using Bayesian method, and the resistivity distribution is reconstructed by the mean estimation of the *a posteriori*. The Markov chain Monte Carlo (MCMC) method with Gibbs sampler is used to sample from posterior density. The simulations on the 2D model show the feasibility of the method.

I. INTRODUCTION

The impedance information of a subject is important in many researches of biomedical engineering. Electrical impedance tomography (EIT) is a useful technique for reconstructing the impedance distribution in the subject from the voltages measured on the surface given the injected current. Since the injected current is of low frequency, the field of EIT can be treated as quasi-static, therefore only the resistivity is considered.

As an inverse problem, EIT imaging is intrinsically non-linear and ill-posed [1] and the prior information must be included to regularize the solutions. The conventional regularizations on resistivity reconstruction assume smooth variation of the solution, which put too much constraint on the possible high resolution in retrieving small object. The "blocky" resistivity is a reasonable model for handling the sharp variations in the real distribution. The prior of "blocky" resistivity profile can be retrieved by the minimal of total variation using Bayesian method where the energy terms of the prior is specified by total variation. The prior distribution is presented as Gibbs distribution according to the random field theory. The resistivity reconstruction problem on a 2D model is solved by Bayesian approach in this paper.

II. METHODS

A. Forward Model

In EIT, given the resistivity distribution ρ in the subject, with the injected current, the boundary voltage u satisfies the Laplace equation

$$\nabla \cdot \rho^{-1} \nabla u = 0 \quad \text{in } \Omega \tag{1}$$

with boundary conditions

$$\begin{cases} u = u_0 & \text{in } \Gamma_1 \\ \rho^{-1} \dfrac{\partial u}{\partial n} = -J_n & \text{in } \Gamma_2 \end{cases} \tag{2}$$

where u_0 is the measured peripheral voltage vector, J_n denotes the current intensities on the boundary, n is the outward normal. Such a forward problem can be solved by Finite Element Method (FEM).

B. Bayesian approach with blocky prior

In Bayesian approach the resistivity distribution ρ and the measured voltage v are assumed to be multivariate random variables with some joint probability $\pi(v,\rho)$. The goal in the Bayesian approach is to solve the posterior $\pi(\rho\,|\,v)$ of the resistivity distribution given measurements v [2]-[4]. Using the Bayes theorem, the posterior is represented as

$$\pi(\rho\,|\,v) \propto \pi(v\,|\,\rho)\pi(\rho) \tag{3}$$

where, the prior depends on the prior assumptions on the ρ, $\pi(v\,|\,\rho)$ is the likelihood function of observation v with the assumed ρ.

Let $U(\rho)$ denote this FEM-based forward solver, considering the additive noise model, the measurement v is

$$v = U(\rho) + N \tag{4}$$

where N is random measurement noise. Furthermore, it is assumed that the random noise is Gaussian with zero mean, with these choices the likelihood density can be written as

$$\pi(v\,|\,\rho) \propto \exp\left(-\frac{1}{2}\left(v - U(\rho)\right)^T C^{-1}\left(v - U(\rho)\right)\right) \tag{5}$$

where C is the covariance of the noise.

Mostly we are interested in finding a "blocky" resistivity distribution [5]-[6]. We choose resistivity profile that has the least total variation from all resistivities that are consistent with the measured data. Based on Random Field theory, the prior can be presented as a Gibbs distribution:

$$\pi(\rho) \propto \exp(-\alpha V_T(\rho)) \tag{6}$$

where, V_T is the total resistivity variation of all elements in the FEM mesh, α represents the strength of the "blocky" conditions, which is determined by experiments.

So the Bayesian model for 2D EIT problem with prior constraints can be presented in the form of

$$\pi(\rho\,|\,v) \propto \exp\left(-\frac{1}{2}\left(\left(v - U(\rho)\right)^T C^{-1}\left(v - U(\rho)\right) + 2\alpha V_T(\rho)\right)\right) \tag{7}$$

Markov chain Monte Carlo (MCMC) method with Gibbs sampler is used to sample from the posterior density.

III. SIMULATION EXPERIMENTS AND RESULTS

A. Simulation model

The simulated 2D circle model is shown in Fig. 1, where the red dots denote the electrodes and the indexes are from 1 to 16. The domain is meshed into triangular elements. The bipolar injected current patterns are adopted and the potential differences are computed among all electrodes with respect to one reference.

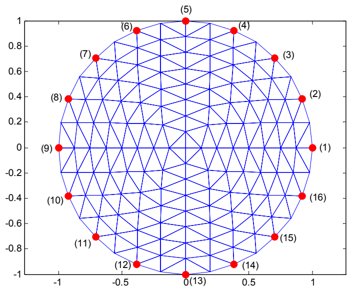


Fig. 1 Two-dimensional simulated FEM model

B. Forward computation results

The forward problem is computed using FEM, and the FEM solution is compared with analysis solution. The resistivity value is assumed as $1\,\Omega \cdot \mathrm{m}$ in the circle domain. Define the relative error of the solution *RDM* as

$$RDM=\sqrt{\frac{\sum_{i=1}^{M}(u_i^{\mathrm{FEM}}-u_i^{\mathrm{analysis}})^2}{\sum_{i=1}^{M}(u_i^{\mathrm{analysis}})^2}} \tag{8}$$

where, u^{FEM} is the FEM solution, u^{analysis} is the analysis solution, M is the number of the electrodes. The *RDM* on the electrodes is 0.41857%.

C. Reconstruction results

For inverse simulation, the background resistivity value is $2\,\Omega \cdot \mathrm{m}$, and the object resistivity value is $3\,\Omega \cdot \mathrm{m}$, as shown in Fig. 2. The noise covariance is assumed to be of the form $C=\sigma^2 I$ in the reconstructions. The confidence parameters σ and α are chosen by experiments. After choosing the different parameters through experiments, we choose $\sigma=10^{-3}$ and $\alpha=200$. The simulated results are shown in Fig. 3, where (a) is the reconstruction image without prior information after sampling 1000 times, and (b) is the reconstruction image with prior information after sampling 600.

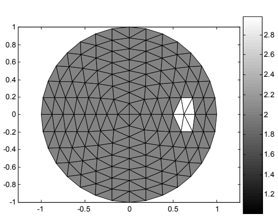
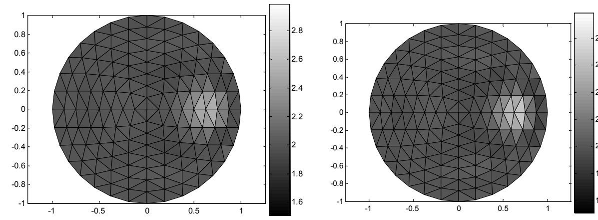


Fig. 2 The resistivity distribution of the 2D simulated model



(a) (b)

Fig. 3 The reconstruction results

From the figures we can see that the object in the image with prior information is clearer and the edge of the object is sharper, and the contrast is better.

IV. CONCLUSION

In this paper, Bayesian approach and MCMC method are used for resistivity reconstruction. The 2D circle model is used for simulation and the FEM is used to solve the forward problem. The simulation results demonstrated that by using "blocky" prior information the reconstructed image has a higher imaging contrast but less sampling operations.

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